

AP Calculus BC

Infinite Series

Direct and Limit Comparison Tests

Name _____

Use the Direct Comparison Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

Converges

2. $\sum_{n=2}^{\infty} \frac{1}{n-1}$

Diverges

3. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

Diverges

4. $\sum_{n=0}^{\infty} \frac{1}{3^n + 1}$

Converges

6. $\sum_{n=0}^{\infty} \frac{3^n}{4^n + 5}$

Converges

7. $\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$

Diverge

8. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$

Converge

9. $\sum_{n=0}^{\infty} \frac{1}{n!}$

Converge

10. $\sum_{n=1}^{\infty} \frac{1}{e^{-n^2}}$

Diverges

11. $\sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$

Diverges

Use the Limit Comparison Test to determine the convergence or divergence of the series.

12. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

Diverges

13. $\sum_{n=1}^{\infty} \frac{2}{3^n - 5}$

Converges

14. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$

Diverges

15. $\sum_{n=3}^{\infty} \frac{3}{\sqrt{n^2 - 4}}$

Diverges

16. $\sum_{n=3}^{\infty} \frac{2n^2 - 1}{3n^5 + 2n + 1}$

Converges

17. $\sum_{n=3}^{\infty} \frac{n+3}{n(n+2)}$

Diverges

18. $\sum_{n=3}^{\infty} \frac{1}{n(n^2 + 1)}$

Converges

19. $\sum_{n=0}^{\infty} \frac{1}{n\sqrt{n^2 + 1}}$

Converges

20. $\sum_{n=3}^{\infty} \frac{n}{(n+1)2^{n-1}}$

Converges

21. $\sum_{n=3}^{\infty} \sin\left(\frac{1}{n}\right)$

Diverges

In exercise 22-28, test for convergence or divergence using each test at least once. Identify which test you used.

(a) nth term Test for divergence

(d) Integral Test

(b) p-test

(e) Direct Comparison Test

(c) Geometric Series Test

(f) Limit Comparison Test

$$22. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$$

Diverges
p-test

$$23. \sum_{n=0}^{\infty} 5 \left(-\frac{1}{5} \right)^n$$

Converges
Geometric Series Test

$$24. \sum_{n=1}^{\infty} \frac{1}{3^n + 1}$$

Converges
Direct Comparison Test

$$25. \sum_{n=4}^{\infty} \frac{1}{3n^2 - 2n - 15}$$

Converges
Limit Comparison Test

$$26. \sum_{n=1}^{\infty} \frac{n}{2n + 3}$$

Diverges
nth term test

$$27. \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}$$

Converges
Integral Test